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ON A SPECIAL CASE OF EQUILIBRIUM OF A FLEXIBLE, INEXTENSIBLE STRING.

By **CHARLES C. STECK**, Durham, New Hampshire.

It is proposed in this paper to discuss the curve of equilibrium of a weightless, flexible, inextensible string, the end points of which are attached to two fixed points, each element of the string being acted upon by a force always normal to an axis which it meets, this force being any function of the distance r of the element from the given axis. A special discussion is made of the case where the force varies inversely as r^2.

The general problem as here stated is suggested by Appell in his *Traité de Mécanique Rationelle,* Vol. I, Chap. VII. In connection with this problem he gives the three following equations which are directly obtainable from the results he has arrived at in his discussion of equilibrium of strings:

$$(1)\quad T\frac{dz}{ds}=c;\quad (2)\quad Tr^2\frac{d\theta}{ds}=k;\quad (3)\quad T=-\int F(r)dr-h=\phi(r),$$

in which T is the tension at any point, $F(r)$ the force, the z-axis the axis to which the force is normal, r and θ the polar co-ordinates in the xy plane, and c, k, h arbitrary constants.

Starting with these three equations and with the equation

$$(4)\qquad ds^2=dr^2+r^2d\theta^2+dz^2,$$

we have, on eliminating T,

$$(5)\qquad k\frac{dr}{rd\theta}=\pm\sqrt{\{r^2[\phi(r)]^2-c^2r^2-k^2\}},$$

$$cr\frac{dr}{dz}=\pm\sqrt{\{r^2[\phi(r)]^2-c^2r^2-k^2\}}, \tag{6}$$

$$r\phi(r)\frac{dr}{dz}=\pm\sqrt{\{r^2[\phi(r)]^2-c^2r^2-k^2\}}. \tag{7}$$

On integrating these equations we obtain

$$\theta-\theta_0=\pm k\int\frac{dr}{r\sqrt{\{r^2[\phi(r)]^2-c^2r^2-k^2\}}}, \tag{8}$$

$$z-z_0=\pm c\int\frac{rdr}{\sqrt{\{r^2[\phi(r)]^2-c^2r^2-k^2\}}}, \tag{9}$$

$$s-s_0=\pm\int\frac{r\phi(r)\,dr}{\sqrt{\{r^2[\phi(r)]^2-c^2r^2-k^2\}}}. \tag{10}$$

Equations (8) and (9) define the curve of equilibrium of the string. Equation (8) is the equation of a cylindrical surface the elements of which are perpendicular to the plane xy. Equation (9) is the equation of a surface of revolution, the axis of revolution being the z-axis. Equations (8), (9), and (10) contain six arbitrary constants which are to be determined from initial conditions.

SPECIAL CASES.

Case I. $c=0,\ k\neq 0$.

From (1) when $c=0$, either $T=0$ or $\frac{dz}{ds}=0$. Since T cannot be zero when $F(r)$ is not zero, $\frac{dz}{ds}$ must be zero; that is, the curve of equilibrium is in a plane perpendicular to the z-axis. Equations (8), (9), and (10) take the following forms:

$$\theta-\theta_0=\pm k\int\frac{dr}{r\sqrt{\{[\phi(r)]^2r^2-k^2\}}}, \tag{11}$$

$$z-z_0=0, \tag{12}$$

$$s-s_0=\pm\int\frac{r\phi(r)\,dr}{\sqrt{\{[\phi(r)]^2r^2-k^2\}}}. \tag{13}$$

Equations (11) and (12) define the curve of equilibrium of a string under the action of a central force.

Case II. $k=0$, $c\neq 0$, $r\neq 0$.

From (2), when $k=0$, either $T=0$ or $r^2\frac{d\theta}{ds}=0$. As in Case I, T cannot be zero, and it follows that $r^2\frac{d\theta}{ds}$ must be zero; that is, $\frac{d\theta}{ds}=0$ or $\theta=\theta_0$. The curve of equilibrium lies in a plane perpendicular to the xy plane and passing through the z-axis. Equations (8), (9), and (10) take the forms

$$\theta-\theta_0=0, \tag{14}$$

$$z-z_0=\pm c\int\frac{dr}{\sqrt{\{[\phi(r)]^2-c^2\}}}, \tag{15}$$

$$s-s_0=\pm\int\frac{\phi(r)\,dr}{\sqrt{\{[\phi(r)]^2-c^2\}}}. \tag{16}$$

Case III. $k=0$, $c=0$, $r\neq 0$.

For this case equations (8), (9), and (10) take the forms,

$$\theta-\theta_0=0, \tag{17}$$

$$z-z_0=0, \tag{18}$$

$$s-s_0=r. \tag{19}$$

The curve of equilibrium is a straight line perpendicular to the z-axis. Since this is true whatever be the force, so long as it is a function of r and r is nowhere zero, it will be unnecessary to consider this case in the discussion which follows and which deals with a special force.

DISCUSSION OF THE CASE WHERE $F(r)=\frac{\mu}{r^2}$, $\mu>0$.

Since we shall deal only with finite forces, it follows that r can nowhere be zero; that is, the string cannot touch the z-axis at any point. Further, since $\mu>0$, we shall have a repulsive force. This constant μ is the value of the force at a unit's distance from the z-axis. Substituting the above value of $F(r)$ in equation (3) and integrating, we have

$$\phi(r)=\frac{\mu}{r}-h=T. \tag{20}$$

Substituting this value of $\phi(r)$ in equations (8), (9), and (10), we get,

$$\theta-\theta_0=\pm k\int\frac{dr}{r\sqrt{\{r^2(h^2-c^2)-2h\mu r+\mu^2-k^2\}}}, \tag{21}$$

$$z-z_0=\pm c\int\frac{rdr}{\sqrt{\{r^2(h^2-c^2)-2h\mu r+\mu^2-k^2\}}}, \tag{22}$$

$$s-s_0=\pm\int\frac{(\mu-hr)dr}{\sqrt{\{r^2(h^2-c^2)-2h\mu r+\mu^2-k^2\}}}. \tag{23}$$

These three indicated integrations can be immediately performed. The following cases must be distinguished.

I. $h^2>c^2$, $\mu^2>k^2$.
  (a) $c^2\neq 0$, $k^2\neq 0$.
  (b) $c^2=0$, $k^2\neq 0$.
  (c) $c^2\neq 0$, $k^2=0$.
II. $h^2>c^2$, $\mu^2=k^2$.
  (a) $c^2\neq 0$.
  (b) $c^2=0$.
III. $h^2>c^2$, $\mu^2<k^2$.
  (a) $c^2\neq 0$.
  (b) $c^2=0$.
IV. $h^2<c^2$, $\mu^2>k^2$.
  (a) $h^2\neq 0$, $k^2\neq 0$.
  (b) $h^2=0$, $k^2\neq 0$.
  (c) $h^2\neq 0$, $k^2=0$.
  (d) $h^2=0$, $k^2=0$.
V. $h^2<c^2$, $\mu^2=k^2$.
  (a) $h^2\neq 0$.
  (b) $h^2=0$.
VI. $h^2<c^2$, $\mu^2<k^2$.
  (a) $h^2\neq 0$.
  (b) $h^2=0$.
VII. $h^2=c^2$, $\mu^2>k^2$.
  (a) $h^2\neq 0$, $k^2\neq 0$.
  (b) $h^2=0$, $k^2\neq 0$.
  (c) $h^2\neq 0$, $k^2=0$.
VIII. $h^2=c^2$, $\mu^2=k^2$.
  (a) $h^2\neq 0$.
  (b) $h^2=0$.
IX. $h^2=c^2$, $\mu^2<k^2$.
  (a) $h^2\neq 0$.
  (b) $h^2=0$.

The following notations are used:

$$R\equiv r^2(h^2-c^2)-2h\mu r+\mu^2-k^2,$$
$$m^2\equiv h^2-c^2 \text{ if } h^2>c^2,$$
$$-m^2\equiv h^2-c^2 \text{ if } h^2<c^2,$$
$$n^2\equiv\mu^2-k^2 \text{ if } \mu^2>k^2,$$
$$-n^2\equiv\mu^2-k^2 \text{ if } \mu^2<k^2,$$
$$\Delta\equiv h^2\mu^2-(h^2-c^2)(\mu^2-k^2).$$

We give here an outline of the method used in determining the constants for Case I (a). The results for this case and for all of the others are put in tabular form at the end, followed by a series of notes by which necessary explanations are made.

DETERMINATION OF THE CONSTANTS FOR CASE I (a).

Initial conditions. Let the initial value of r be that value which makes $R=0$. Call this value r_1. When $r=r_1$, let $\theta=0$, $z=0$, $s=0$, $T=T_1$.

Interpretation of initial conditions. Returning to equations (1) and (2), we see that when $R=0$, since $k\neq 0$ and $c\neq 0$, $\frac{1}{r}\frac{dr}{d\theta}=0$ and $\frac{dr}{dz}=0$. Now $\frac{1}{r}\frac{dr}{d\theta}$ is the cotangent of the angle between the radius vector and the tangent to the guiding curve of the cylinder at any point. We shall call this angle ϕ. $\frac{dr}{dz}$ gives the slope of the tangent to the meridian section of the surface of revolution at the same point. Call this angle ψ. With the initial conditions just imposed $\phi_1=\frac{1}{2}\pi$ and $\psi_1=0$.

From (1) it is evident that c is the value of the tension when $\frac{dz}{ds}=1$; *i. e.*, the component of the tension parallel to the z-axis; call it T_0. c may always be taken positively if we measure s in such a direction that z and s increase simultaneously since T is essentially positive. The constant h may be found from equation (20). We have, on imposing initial conditions

$$h=\frac{\mu}{r_1}-T_1.$$

The constant k is found from the equation $R=0$. Solving this equation for k^2 we get

$$k^2=r_1^2(T_1^2-T_0^2).$$

The values of θ_0, z_0, and s_0 are

$$\theta_0=0,\quad z_0=0,\quad s_0=0.$$

The corresponding values of m^2, n^2, and Δ are

$$m^2=h^2-c^2=\frac{1}{r_1^2}[\mu^2-2\mu r_1T_1+r_1^2(T_1^2-T_0^2)],$$

$$n^2=\mu^2-k^2=\mu^2-r_1^2(T_1^2-T_0^2),$$

$$\Delta=[\mu T_1-r_1(T_1^2-T_0^2)]^2.$$

Case			r_1	Equation of Cylinder	θ_0	Equation of Surface of Revolu	z_0	Equation of Length of Arc	s_0	Notes
I $h^2>c^2$ $\mu^2>k^2$	a	$c^2\neq0$ $k^2\neq0$	$\frac{h\mu-\sqrt{\Delta}}{m^2}$	$r=\frac{n^2}{h\mu+\sqrt{\Delta}\cosh^{-1}[\pm\frac{n}{k}(\theta-\theta_0)]}$	0	$z-z_0=\pm c\left\{\frac{\sqrt{R}}{m^2}+\frac{h\mu}{m^3}\cosh^{-1}\left(\frac{h\mu-m^2r}{\sqrt{\Delta}}\right)\right\}$	0	$s-s_0=\pm\left\{\frac{\mu}{m}\left(1-\frac{h^2}{m^2}\right)\cosh^{-1}\left(\frac{h\mu-m^2r}{\sqrt{\Delta}}\right)-\frac{h\sqrt{R}}{m^2}\right\}$	0	A B
	b	$c^2=0$ $k^2\neq0$	$\frac{h\mu-\sqrt{\Delta}}{m^2}$	$r=\frac{n^2}{h\mu+\sqrt{\Delta}\cosh^{-1}[\pm\frac{m}{k}(\theta-\theta_0)]}$	0	$z-z_0=0$	0	$s-s_0=\mp\frac{\sqrt{R}}{h}$	0	A B
	c	$c^2\neq0$ $k^2=0$	$\frac{h\mu-\sqrt{\Delta}}{m^2}$	$\theta-\theta_0=0$	0	$z-z_0=\pm c\left\{\frac{\sqrt{R}}{m^2}+\frac{h\mu}{m^3}\cosh^{-1}\left(\frac{h\mu-m^2r}{\sqrt{\Delta}}\right)\right\}$	0	$s-s_0=\pm\left\{\frac{\mu}{m}\left(1-\frac{h^2}{m^2}\right)\cosh^{-1}\left(\frac{h\mu-m^2r}{\sqrt{\Delta}}\right)-\frac{h\sqrt{R}}{m^2}\right\}$	0	A B
II $h^2>c^2$ $\mu^2=k^2$	a	$c^2\neq0$	$\frac{k\cot\phi_1}{c\tan\psi_1}$	$r=\frac{2\mu h}{m^2-h^2(\theta-\theta_0)^2}$	$\pm\frac{c}{h}\tan\psi_1$	$z-z_0=\pm c\left\{\frac{\sqrt{R}}{m^2}+\frac{h\mu}{m^3}\cosh^{-1}\left(\frac{h\mu-m^2r}{h\mu}\right)\right\}$	$\pm c\left\{\frac{k\cot\phi_1}{m^3}+\frac{h\mu}{m^3}\cosh^{-1}\left(\frac{h\mu-m^2r_1}{h\mu}\right)\right\}$	$s-s_0=\pm\left\{\frac{\mu}{m}\left(1-\frac{h^2}{m^2}\right)\cosh^{-1}\left(\frac{h\mu-m^2r}{h\mu}\right)-\frac{h\sqrt{R}}{m^2}\right\}$	$\mp\left\{\frac{\mu}{m}\left(1-\frac{h^2}{m^2}\right)\cosh^{-1}\left(\frac{h\mu-m^2r_1}{h\mu}\right)-\frac{h\sqrt{R}}{m^3}\right\}$	C D E
	b	$c^2=0$	$\frac{\mu}{h}(1\pm\cos\phi)$	$r=\frac{2\mu}{h[1-(\theta-\theta_0)^2]}$	$\pm\frac{k\cot\phi_1}{hr_1}$	$z-z_0=0$	0	$s-s_0=\mp\frac{\sqrt{R}}{h}$	$s_0=\pm\frac{k\cot\phi_1}{h}$	C D
III $h^2>c^2$ $\mu^2<k^2$	a	$c^2\neq0$	$\frac{h\mu+\sqrt{\Delta}}{m^2}$	$r=-\frac{n^2}{h\mu-\sqrt{\Delta}\cos\frac{n}{k}\theta}$	$\frac{k}{n}\cdot\frac{\pi}{2}$	$z-z_0=\pm c\left\{\frac{\sqrt{R}}{m^2}+\frac{h\mu}{m^3}\cosh^{-1}\left(\frac{m^2r-h\mu}{\sqrt{\Delta}}\right)\right\}$	0	$s-s_0=\pm\left\{\frac{\mu}{m}\left(1-\frac{h^2}{m^2}\right)\cosh^{-1}\left(\frac{m^2r-h\mu}{\sqrt{\Delta}}\right)-\frac{h\sqrt{R}}{m^2}\right\}$	0	C D F
	b	$c^2=0$	$\frac{h\mu+\sqrt{\Delta}}{h^2}$	$r=\frac{-n^2}{h\mu-\sqrt{\Delta}\cos\frac{n}{k}\theta}$	$\frac{k}{n}\cdot\frac{\pi}{2}$	$z-z_0=0$	0	$s-s_0=\mp h\frac{\sqrt{R}}{m^2}$	0	C D F
IV $h^2<c^2$ $\mu^2>k^2$	a	$h^2\neq0$ $k^2\neq0$	$\frac{-h\mu+\sqrt{\Delta}}{m^2}$	$r=\frac{n^2}{h\mu+\sqrt{\Delta}\cosh[\pm\frac{n}{k}(\theta-\theta_0)]}$	0	$z-z_0=\pm c\left\{\frac{\sqrt{R}}{m^2}+\frac{h\mu}{m^3}\sin^{-1}\left(\frac{m^2r+h\mu}{\sqrt{\Delta}}\right)\right\}$	$\mp\frac{ch\mu}{m^3}\cdot\frac{\pi}{2}$	$s-s_0=\pm\left\{\frac{\mu}{m}\left(1+\frac{h^2}{m^2}\right)\sin^{-1}\left(\frac{m^2r-h\mu}{\sqrt{\Delta}}\right)+\frac{h\sqrt{R}}{m^2}\right\}$	$\mp\frac{\mu}{m}\left(1+\frac{h^2}{m^2}\right)\frac{\pi}{2}$	A
	b	$h^2=0$ $k^2\neq0$	$\frac{n}{m}$	$r=\frac{n}{m\cosh[\pm\frac{n}{k}(\theta-\theta_0)]}$	0	$r^2+(z-z_0)^2=r_1^2$	0	$s-s_0=\pm\left\{\frac{\mu}{m}\sin^{-1}\frac{mr}{n}\right\}$	0	A
	c	$h^2\neq0$ $k^2=0$	$\frac{-h\mu+\sqrt{\Delta}}{m^2}$	$\theta-\theta_0=0$	0	$z-z_0=\pm c\left\{\frac{\sqrt{R}}{m^2}+\frac{h\mu}{m^3}\sin^{-1}\left(\frac{m^2r+h\mu}{\sqrt{\Delta}}\right)\right\}$	$\mp\frac{ch\mu}{m^3}\cdot\frac{\pi}{2}$	$s-s_0=\pm\left\{\frac{\mu}{m}\left(1+\frac{h^2}{m^2}\right)\sin^{-1}\left(\frac{m^2r-h\mu}{\sqrt{\Delta}}\right)+\frac{h\sqrt{R}}{m^2}\right\}$	$\mp\frac{\mu}{m}\left(1+\frac{h^2}{m^2}\right)\frac{\pi}{2}$	A
	d	$h^2=0$ $k^2=0$	$\frac{\mu}{c}$	$\theta-\theta_0=0$	0	$\frac{r^2}{\mu^2}+\frac{(z-z_0)^2}{c^2\mu^2}=1$	0	$s-s_0=\pm\left\{\frac{\mu}{c}\sin^{-1}\frac{c}{\mu}r\right\}$	0	A
V $h^2<c^2$ $\mu^2=k^2$	a	$h^2\neq0$	$\frac{-2h\mu}{m^2}$	$r=\frac{-2h\mu}{h^2(\theta-\theta_0)^2+m^2}$	0	$z-z_0=\pm c\left\{\frac{\sqrt{R}}{m^2}+\frac{h\mu}{m^3}\sin^{-1}\left(\frac{m^2r+h\mu}{\sqrt{\Delta}}\right)\right\}$	$\mp\frac{ch\mu}{m^3}\cdot\frac{\pi}{2}$	$s-s_0=\pm\left\{\frac{\mu}{m}\left(1+\frac{h^2}{m^2}\right)\sin^{-1}\left(\frac{m^2r+h\mu}{\sqrt{\Delta}}\right)+\frac{h\sqrt{R}}{m^2}\right\}$	$\mp\frac{\mu}{m}\left(1+\frac{h^2}{m^2}\right)\frac{\pi}{2}$	C
	b	$h^2=0$								G
VI $h^2<c^2$ $\mu^2<k^2$	a	$h^2\neq0$	$\frac{-h\mu-\sqrt{\Delta}}{m^2}$	$r=-\frac{n^2}{h\mu-\sqrt{\Delta}\cos\frac{n}{k}\theta}$	$\frac{k}{n}\cdot\frac{\pi}{2}$	$z-z_0=\pm c\left\{\frac{\sqrt{R}}{m^2}+\frac{h\mu}{m^3}\sin^{-1}\left(\frac{m^2r-h\mu}{\sqrt{\Delta}}\right)\right\}$	$\pm\frac{ch\mu}{m^3}\cdot\frac{\pi}{2}$	$s-s_0=\pm\left\{\frac{\mu}{m}\left(1+\frac{h^2}{m^2}\right)\sin^{-1}\left(\frac{m^2r+h\mu}{\sqrt{\Delta}}\right)+\frac{h\sqrt{R}}{m^2}\right\}$	$\mp\frac{\mu}{m}\left(1+\frac{h^2}{m^2}\right)\frac{\pi}{2}$	C F
	b	$h^2=0$								G
VII $h^2=c^2$ $\mu^2>k^2$	a	$h^2\neq0$ $k^2\neq0$	$\frac{k\cot\phi_1}{c\tan\psi_1}$	$r=\frac{-n^2}{h\mu}\cdot\frac{1}{1\cosh[\pm\frac{n}{k}(\theta-\theta_0)]}$	$\pm\frac{k}{n}\log\left(\frac{k\cot\phi_1-n}{k\cot\phi_1+n}\right)$	$(z-z_0)^2=\frac{1}{9h^2\mu^4}[n^6-3h^2\mu^2n^2r^2-2h^3\mu^3r^3]$	$\mp\frac{(n^2+h\mu r_1)}{3\mu^2}r_1$	$s-s_0=\frac{\pm n^2+h\mu r-3\mu^2}{3h\mu^2}\sqrt{R}$	$\mp\frac{(2\mu^2-k^2-h\mu r_1)}{3\mu^2}r_1$	C E
	b	$h^2=0$ $k^2\neq0$	z_1	$r=e^{\pm\frac{n}{k}(\theta-\theta_0)}$	0	$z-z_0=0$	0	$s-s_0=\pm\frac{(\mu r-hr^2)}{2}$	$\mp\left(\frac{\mu-h}{2}\right)$	C
	c	$h^2\neq0$ $k^2=0$	$\frac{-\mu}{h}$	$\theta-\theta_0=0$	0	$(z-z_0)^2=\frac{1}{9h^2\mu}(\mu^3-3h^2\mu r^2-2h^3r^3)$	0	$s-s_0=\frac{\pm n^2+h\mu r-3\mu^2}{3h\mu^2}\sqrt{R}$	$\mp\frac{(2\mu^2-h\mu r_1)}{3\mu^2}r_1$	C
VIII $h^2=c^2$ $\mu^2=k^2$	a	$h^2\neq0$	$\frac{k\cot\phi_1}{c\tan\psi_1}$	$r=\frac{-2k^2}{\mu h(\theta-\theta_0)^2}$	$\frac{-2\mu}{\sqrt{-2\mu hr_1}}$	$(z-z_0)^2=\frac{2}{9}\frac{c}{\mu}r^3$	$\frac{\mp\frac{2}{3}r_1^{\frac{3}{2}}h}{\sqrt{-2\mu h}}$	$(s-s_0)^2=\frac{-2r}{9h\mu}(3\mu-hr)^2$	$\frac{\mp2r_1^{\frac{1}{2}}(3\mu-hr_1)}{3\sqrt{-2\mu h}}$	C
	b	$h^2=0$	r_1	$r=r_1$		$z-z_0=0$	0			C H
IX $h^2=c^2$ $\mu^2<k^2$	a	$h^2\neq0$	$\frac{-n^2}{2h\mu}$	$r=\frac{-n^2}{\mu h}\cdot\frac{1}{1+\cos\frac{n}{k}(\theta-\theta_0)}$	0	$(z-z_0)^2=\frac{1}{9\mu^4h^2}(2h^3\mu^3r^3+3h^2\mu^2n^2r^2-n^6)$	0	$s-s_0=\frac{\pm h\mu r-n^2-3\mu^2}{3h\mu^2}\sqrt{R}$	$\mp\frac{(2\mu^2-k^2-h\mu r_1)}{3\mu^2}r_1$	C F
	b	$h^2=0$								G

NOTES ON PRECEDING TABLE.

A. $h>0$.

B. $m<0$; $n>0$.

C. $h<0$.

D. $m>0$; $n<0$.

E. Since the value of r_1 used in Case I has become zero in this case, and since this contradicts the hypothesis that $F(r)$ is finite, it follows that $\phi_1 \neq \frac{1}{2}\pi$, $\psi_1 \neq 0$. The initial value of r cannot be a root of the equation $R=0$. When $r=r_1$, let $\phi=\phi_1 \neq \frac{1}{2}\pi$, $\psi=\psi_1 \neq 0$. With other initial conditions as before, we obtain for k the value

$$k=r_1 T_0 \frac{\tan \psi_1}{\cot \phi_1}.$$

F. In determining the equation of the cylinder for this case we have nsed the positive sign of equation (8) only. The negative sign gives a similar curve.

G. For $h=0$ the expression $\sqrt{R}$ becomes imaginary for all values of r.

H. To determine the curve of equilibrium for this case we have from equation (5), since $R=0$,

$$\frac{k}{r}\frac{dr}{d\theta}=0.$$

Since $k \neq 0$ and $r \neq 0$, we obtain on integration, $r=\text{constant}=r_1$.

Graphs of the guiding curves of the cylinder and a meridian section of the surface of revolution have been made for each case. Since there are so many cases to consider a reproduction of these graphs is not convenient here. In regard to the curve of equilibrium, the prevailing type is a spiral with the z-axis as the line about which it winds.